Excited heavy baryon masses in HQET QCD sum rules

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We construct the interpolating currents for the fourteen p-wave excited heavy baryons in the framework of heavy quark effective theory (HQET). At the leading order in the $1/m_Q$ expansion these interpolating currents do not mix different states with the same J^P with each other. We use QCD sum rule to calculate the masses of the p-wave excited J= $\frac{1}{2}$ spin doublet heavy baryons in the $m_Q \to \infty$ limit. We discuss some implications of our results.

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I. INTRODUCTION

In the past years important progress has been made in the field of heavy baryons. Most of the ground state charmed baryons have been found experimentally while among the bottomed baryons only Λ_b is established [1]. The lowest lying orbitally excited states of Λ_c have been observed by ARGUS [2], E687 [3] and CLEO [4] collaborations in the decay channel $\Lambda_c\pi\pi$. The decay width of Λ_{c1} with $J^P = \frac{1}{2}^-$ is $3.6^{+2.0}_{-1.3}$ MeV [1]. For the state Λ_{c1} with $J^P = \frac{3}{2}^-$ only an upper limit of 1.9 MeV is set [1]. Preliminary evidence of excited baryon with charm and strange quarks in the decay channel $\Xi^+\pi\pi$ was reported by CLEO collaboration [5]. Its width is less than 2.4 MeV.

Theoretically the heavy quark effective theory (HQET) [6] provides a consistent framework to study baryons containing one heavy quark in terms of $1/m_Q$ expansion with m_Q the heavy quark mass. Heavy baryon mass m_B can be expanded as $m_B = m_Q + \bar{\Lambda} + \mathcal{O}(1/m_Q)$ where $\bar{\Lambda}$ is the heavy baryon binding energy in the leading order. Corrections at higher orders of the heavy quark expansion can be included consistently. However in order to extract $\bar{\Lambda}$ we have to turn to other nonperturbative methods such as lattice gauge theory, QCD sum rules (QSR) [7] etc for guidance. In this work we will use the latter method to calculate the masses, or equivalently, the binding energies of the p-wave orbitally excited heavy baryons in the leading order. The spectrum of ground state heavy baryons have been studied with HQET using QSR in Refs. [8–10]. Recently the binding energy of the lowest two states of orbitally excited charmed baryons $\Lambda_{c1}, \Lambda^*_{c1}$ has been calculated [13,14]. However, the interpolating currents creating all the p-wave excited states have not been given. In this work we construct the interpolating currents for the fourteen p-wave excited heavy baryons in the framework of heavy quark effective theory (HQET) and present a complete calculation of the spectrum of the p-wave excited spin 1/2 doublet heavy baryons at the leading order in the $1/m_Q$ expansion. Our calculation confirms earlier observation that the gluon condensates are of opposite sign to the leading perturbative term [13].

Excited heavy baryons are natural labs to test theoretical frameworks like HQET and explore the strong interaction dynamics which simplifies in the limit of $m_Q \to \infty$. Analyses of various decays can also be used to extract some basic parameters of standard model. There are many theoretical papers in the recent years. The strong decays of the excited heavy baryons were studied within the framework of heavy baryon chiral perturbation theory [15,16], various versions of quark model [18–22,25,24], and light cone QCD sum rules [13]. The electromagnetic decays were analyzed using a bound state picture in [17], quark models in [23,24] and light cone QCD sum rules [13]. Our construction of proper interpolating currents and calculation of leading order binding energy will faciliate the future study of pionic, electromagnetic and semileptonic decays of these excited states within the framework of QSR.

II. INTERPOLATING CURRENTS

For a heavy baryon composed of a heavy quark Q and light freedom degrees, i.e., a light diquark system (qq), the spin-parity j^P of the light diquark system is conserved in the $m_Q \to \infty$ limit. Given j^P, one has a degenerate heavy baryon spin doublet with $J^P = (j \pm 1/2)^P$. The heavy quark symmetry structure of heavy baryons is completely determined by the spin- parity j^P of the light diquark system.

A systematic classification of heavy baryon states by constituent approach has been presented [11]. For p-wave excited heavy baryons, there are two independent orbital angular momenta l_k and l_K corresponding to the two independent momenta k and K which we take to be the relative momenta k=$\frac{1}{2}(p_1 - p_2)$ and K=$\frac{1}{2}(p_1 + p_2 - 2p_3)$ where p_1 and p_2 are the light quark momenta and p_3 the heavy quark momentum. The choice (k, K) basis has two advantages. First, with such a choice the physical meaning of l_k and l_K is transparent: the k-orbital angular momentum l_k describes relative motion of the two light quarks, and the K-orbital angular momentum l_K describes orbital motion of the center of mass of the two light quarks relative to the heavy quark. Second, the (k, K) basis allows one to classify the diquark states in terms of SU($2N_f$)$\otimes$O(3) representations [11] where N_f is the number of light flavors. We assume N_f=2 in this paper and generization to the N_f=3 case is straightforward.

Symmetric property	j^P	J^P	State
		$\frac{1}{2}^-$	
$[q_1q_2]$:	1^- $\nearrow$ $\searrow$		$\{\Lambda_{QK1}\}$
		$\frac{3}{2}^-$	
$\{q_1q_2\}$:	$0^- \rightarrow$	$\frac{1}{2}^-$	Σ_{QK0}
		$\frac{1}{2}^-$	
	1^- $\nearrow$ $\searrow$		$\{\Sigma_{QK1}\}$
		$\frac{3}{2}^-$	
		$\frac{3}{2}^-$	
	2^- $\nearrow$ $\searrow$		$\{\Sigma_{QK2}\}$
		$\frac{5}{2}^-$	.

TABLE I. The p-wave heavy baryon states, with l_k=0 and l_K=1.

According to the analysis in ref. [11], the p-wave excitation with l_k=0 and l_K=1 belongs to the representation $10 \otimes 3_K$ of SU(4)$\otimes$ O(3). Under the $SU(2)_{spin} \otimes SU(2)_{flavor}$ the 10 decomposes into $1 \otimes 1$ and $3 \otimes 3$. The spin 0 and 1 pieces of the 10 couple with l_K=1 to give $j^P = 1^-$ state with flavor anti-symmetric (Λ-type) and $j^P = 0^-, 1^-, 2^-$ states with flavor symmetric (Σ-type), respectively. The total angular momentum j of a diquark with definite parity P couples with the spin of the heavy quark finally to give the p-wave heavy baryon with spin-parity J^P states which are listed in Table 1. Similar analyses apply to the p-wave excitations with l_k=1 and l_K=0 and results are listed in Table 2.

Thus we have total fourteen p-wave heavy baryon states with negative parity of which seven states are Λ-type and the others Σ-type, corresponding to $[q_1q_2] = q^T \tau_A q$ with $q^T = (q_1^T, q_2^T)$ and $\{q_1q_2\} = q^T \tau_i q$ (i=1,2,3) (for the definition of τ_B , B=1,2,3,A, see below), respectively. We denote them by B_{Qpj} with B=Λ, Σ, Q=b,c, p=k,K and j=0,1,2.

An important step to carry out QCD sum rule analysis of Green functions is to construct appropriate interpolating currents which create corresponding heavy baryon states. For the above fourteen p-wave states, we propose to use

$$\bar{j}^{B,p,j,J}_{\rho_1\ldots\rho_J} \equiv j^{\dagger\, B,p,j,J}_{\rho_1\ldots\rho_J} \gamma^0 \tag{1}$$

as such interpolating currents. Here

Symmetric property	j^P	J^P	State
$[q_1q_2]$:	$0^- \rightarrow$	$\frac{1}{2}^-$	Λ_{QK0}
		$\frac{1}{2}^-$	
	1^- $\nearrow$ $\searrow$		$\{\Lambda_{QK1}\}$
		$\frac{3}{2}^-$	
		$\frac{3}{2}^-$	
	2^- $\nearrow$ $\searrow$		$\{\Lambda_{QK2}\}$
		$\frac{5}{2}^-$	.
$\{q_1q_2\}$:		$\frac{1}{2}^-$	
	1^- $\nearrow$ $\searrow$		$\{\Sigma_{QK1}\}$
		$\frac{3}{2}^-$	

TABLE II. The p-wave heavy baryon states, with l_k=1 and l_K=0.

$$j^{B,p,j,J}_{\rho_1\ldots\rho_J}(x) = \epsilon^{abc}(q_a^T(x)\tau_B(a+b\not{v})\phi_p^{\mu_1\ldots\mu_j}Cq_b(x))\Gamma^J_{\mu_1\ldots\mu_j;\rho_1\ldots\rho_J}h_{v,c}(x), \tag{2}$$

where

$$\phi_p^{\mu_1\ldots\mu_j} = \phi^{\mu_1\ldots\mu_j;\nu}(v)D^p_\nu, \qquad D^p_\nu = \begin{cases} \dfrac{\partial}{\partial x^\nu} - igA_\nu(x) & \text{for p=k} \\ \dfrac{\partial}{\partial y^\nu} - igA_\nu(y) & \text{for p=K} \end{cases} \tag{3}$$

with x and y the relative coordinates between light quarks and between the center of mass of light quark system and the heavy quark respectively. $D_\mu(x)$ operates on a light quark field q and $D_\mu(y)$ on the effective heavy quark field h_v (see Table 3 for details).

$$\Gamma^J_{\mu_1\ldots\mu_j;\rho_1\ldots\rho_J} = \begin{cases} \Gamma^{j+1/2}_{\mu_1\ldots\mu_j;\rho_1\ldots\rho_j} & \text{for J=j+}\frac{1}{2} \\ \Gamma^{j-1/2}_{\mu_1\ldots\mu_j;\rho_1\ldots\rho_{j-1}} & \text{for J=j-}\frac{1}{2} \end{cases} \tag{4}$$

consists of Dirac γ matrices and the covariant derivative, and τ_B (B=1,2,3,A) are defined by

$$\tau_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \tau_2 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \tau_3 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \tau_A = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{5}$$

States	$\phi_p^{\mu_1\cdots\mu_j}$	j^P	$\Gamma^J_{\mu_1\ldots\mu_j;\rho_1\ldots\rho_J}$	J^P
p=k, i.e., $l_k=1, l_K=0$				
$\{\Sigma_{Qk1}\}$	$\gamma_5 D_t^{\mu_1}(x)$	1^-	$\gamma_{t\mu_1}\gamma_5$	$\frac{1}{2}^-$
			$\Gamma_{\mu_1\rho_1}$	$\frac{3}{2}^-$
$\{\Lambda_{Qk0}\}$	$\gamma_t^\mu D_{t\mu}(x)$	0^-	I	$\frac{1}{2}^-$
$\{\Lambda_{Qk1}\}$	$\epsilon^{\mu_1\nu\sigma\rho}\gamma_{t\nu}D^\sigma_t(x)v^\rho$	1^-	$\gamma_{t\mu_1}\gamma_5$	$\frac{1}{2}^-$
			$\Gamma_{\mu_1\rho_1}$	$\frac{3}{2}^-$
$\{\Lambda_{Qk2}\}$	$\{\gamma_t^{\mu_1}D_t^{\mu_2}(x)\}_0$	2^-	$\gamma_5\gamma_{t\{\mu_1}\Gamma_{\mu_2\}\rho_1}$	$\frac{3}{2}^-$
			$\Gamma_{\rho_1\{\mu_1}\Gamma_{\mu_2\}0\rho_2}$	$\frac{5}{2}^-$
p=K, i.e., $l_k=0, l_K=1$				
$\{\Lambda_{QK1}\}$	$\gamma_5 D_t^{\mu_1}(y)$	1^-	$\gamma_{t\mu_1}\gamma_5$	$\frac{1}{2}^-$
			$\Gamma_{\mu_1\rho_1}$	$\frac{3}{2}^-$
$\{\Sigma_{QK0}\}$	$\gamma_t^\mu D_{t\mu}(y)$	0^-	I	$\frac{1}{2}^-$
$\{\Sigma_{QK1}\}$	$\epsilon^{\mu_1\nu\sigma\rho}\gamma_{t\nu}D^\sigma_t(y)v^\rho$	1^-	$\gamma_{t\mu_1}\gamma_5$	$\frac{1}{2}^-$
			$\Gamma_{\mu_1\rho_1}$	$\frac{3}{2}^-$
$\{\Sigma_{QK2}\}$	$\{\gamma_t^{\mu_1}D_t^{\mu_2}(y)\}_0$	2^-	$\gamma_5\gamma_{t\{\mu_1}\Gamma_{\mu_2\}\rho_1}$	$\frac{3}{2}^-$
			$\Gamma_{\rho_1\{\mu_1}\Gamma_{\mu_2\}0\rho_2}$	$\frac{5}{2}^-$

TABLE III. The explicit expressions of $\phi_p^{\mu_1\cdots\mu_j}$ and $\Gamma^J_{\mu_1\ldots\mu_j;\rho_1\ldots\rho_J}$ for the p-wave states. Here I is the 4×4 unit matrix and $\Gamma_{\mu\nu} = -\frac{1}{3}(g_{t\mu\nu} + \gamma_{t\nu}\gamma_{t\mu})$ with $\gamma_t^\mu = \gamma^\mu - v^\mu\not{v}$. $D_\mu(x)$ operates on a light quark field q and $D_\mu(y)$ the effective heavy quark field h_v.

The matrices τ_B describe the flavor structures of the diquark inside a heavy baryon and satisfy

$$tr(\tau_B \tau^{\dagger}_{B'}) = \delta_{BB'}, \qquad B, B^{'} = 1, 2, 3, A. \tag{6}$$

The explicit expressions of $\phi_p^{\mu_1 \ldots \mu_j}$ and $\Gamma^J_{\mu_1 \ldots \mu_j;\rho_1 \ldots \rho_J}$ for the p-wave states are listed in Table 3. Let $\mid B, p, j, J >$ be a p-wave excited heavy baryon state with quantum numbers j, J and definite flavor type B and orbit angular momentum type p in the $m_Q \to \infty$ limit. Because we limit ourselves only to p-wave excited states which all have negative parity in this work, we omit the parity index. We have

$$< 0 \mid j^{B,p,j,J}_{\rho_1 \ldots \rho_J}(0) \mid B^{'}, p^{'}, j^{'}, J^{'} >= \delta_{BB'} \delta_{pp'} \delta_{jj'} \delta_{JJ'} f_{j,J} u_{\rho_1 \ldots \rho_J}. \tag{7}$$

In order to verify eq.(7) an easy way is to choose the gauge $A_\mu(x_0) = 0$ without loss of generality. In the gauge D_μ(x) is reduced to $\frac{\partial}{\partial x^\mu}$ and the Bethe-Salpeter(BS) wave function of a p-wave excited heavy baryon is defined by

$$\begin{aligned} \chi^{\alpha\beta\gamma;ii'}_{B,p,j,J}(x_1, x_2, x_3) &= < 0 \mid T(q^i_\alpha(x_1) q^{i'}_\beta(x_2) Q_\gamma(x_3)) \mid B, p, j, J > \\ &= e^{-iP \cdot Y} \chi^{\alpha\beta\gamma;ii'}_{B,p,j,J}(x, y) \\ Y = x_1 + x_2 + x_3, \qquad & x = (x_1 - x_2), \qquad y = (x_1 + x_2 - 2x_3)/3 \end{aligned} \tag{8}$$

with α, β, γ being Dirac indices and P the momentum of baryon. Here and hereafter colour indices are suppressed for the sake of simplicity. As shown in ref. [11], in the $m_Q \to \infty$ limit, the BS wave function can be written as

$$\chi^{\alpha\beta\gamma;ii'}_{B,p,j,J}(x, y) = \tau^{\dagger ii'}_B C \bar{\phi}^{\mu_1 \ldots \mu_j}_{p\alpha\beta}(v, x, y)(1 + \not{v}) f(x, y) \Psi^{J,\gamma}_{\mu_1 \ldots \mu_j}, \tag{9}$$

where f(x,y)=$f(x_t^2, y_t^2, x_l, y_l)$ is a Lorentz scalar function of z_t^2 and z_l (z=x,y), $z_l = z \cdot v$, $z_t = z - z_l v$, and $\Psi^J_{\mu_1,\ldots,\mu_j}$ is given by

$$\begin{aligned} \Psi^{\frac{1}{2}} &= u, \\ \Psi^{\frac{1}{2}}_{\mu_1} &= \gamma_{t\mu_1} \gamma_5 u, \\ \Psi^{\frac{3}{2}}_{\mu_1} &= u_{\mu_1}, \\ \Psi^{\frac{3}{2}}_{\mu_1\mu_2} &= \gamma_5 \gamma_{t\{\mu_1} u_{\mu_2\}_0}, \\ \Psi^{\frac{5}{2}}_{\mu_1\mu_2} &= u_{\mu_1\mu_2}, \end{aligned} \tag{10}$$

where the notation $\{\mu_1\mu_2\}_0$ implies symmetrization and tracelessness, $\gamma_t^\mu = \gamma^\mu - v^\mu \not{v}$ with v^μ is the velocity of the heavy baryon, u and $u_{\mu_1 \ldots \mu_j}$ are the usual Dirac spinor and the Rarita-Schwinger spinor respectively. The latter satisfies

$$\gamma^{\mu_i} u_{\mu_1 \ldots \mu_i \ldots \mu_j} = 0, \qquad v^{\mu_i} u_{\mu_1 \ldots \mu_i \ldots \mu_j} = 0 \tag{11}$$

and are the symmetric and traceless. Therefore, from eqs. (2) and (9), we have

$$
\begin{aligned}
< 0 \mid j^{B,p,j,J}_{\rho_1\ldots\rho_J;\gamma}(x_0) \mid B^{'}, p^{'}, j^{'}, J^{'} > &= \tau^{ii'}_B (a+b\not{v}) \phi^{\mu_1\ldots\mu_j}_{p\ \ \alpha\beta} C(\Gamma^J_{\mu_1\ldots\mu_j;\rho_1\ldots\rho_J})^{\gamma\lambda} \\
&\quad \chi^{\beta\alpha\lambda;i'i}_{B'p',j',J'}(x_0 + \frac{3y}{2} + \frac{x}{2}, x_0 + \frac{3y}{2} - \frac{x}{2}, x_0) \mid_{x=y=0} \\
&= tr(\tau_B \tau^{\dagger}_{B'}) \ \ tr(\phi^{\mu_1\ldots\mu_j}_p \bar{\phi}^{\nu_1\ldots\nu_j}_{p'}) \ \ (a+b) \ \ f \\
&\quad (\Gamma^J_{\mu_1\ldots\mu_j;\rho_1\ldots\rho_J} \Psi^{J^{'}}_{\nu_1\ldots\nu_{j'}})^{\gamma} \mid_{x=y=0} \quad e^{-i\bar{\Lambda} v\cdot x_0}.
\end{aligned} \tag{12}
$$

Using

$$
\frac{\partial^n f(x_t^2, y_t^2, x_l, y_l)}{\partial z_t^{\mu_1} \ldots \partial z_t^{\mu_n}} \mid_{x_t=y_t=0} = 0 \qquad for \quad n = odd, \tag{13}
$$

and

$$
\frac{\partial^2 f(x_t^2, y_t^2, x_l, y_l)}{\partial z_t^{\mu} \partial z_t^{'\nu}} \mid_{x_t=y_t=0} = \delta_{z_t z_t^{'}} g^{\mu\nu} 2 \frac{\partial^2 f}{\partial z_t^2} \mid_{z_t=0} \tag{14}
$$

with z, $z^{'}$=x, y, one has

$$
\begin{aligned}
tr(\phi^{\mu_1\ldots\mu_j}_p \bar{\phi}^{\nu_1\ldots\nu_{j'}}_{p'}) f &= tr(\phi^{\mu_1\ldots\mu_j;\mu} \bar{\phi}^{\nu_1\ldots\nu_{j'};\mu}) \delta_{pp'} 2 \frac{\partial^2 f}{\partial z_t^2} \mid_{z_t=0} \\
&= \delta_{jj'} \delta_{pp'} G^{\mu_1\ldots\mu_j;\nu_1\ldots\nu_j} 2 \frac{\partial^2 f}{\partial z_t^2} \mid_{z_t=0},
\end{aligned} \tag{15}
$$

where

$$
\begin{aligned}
G &= 1 \qquad for \quad j = 0, \\
G^{\mu\nu} &= -g_t^{\mu\nu} \equiv g^{\mu\nu} - v^{\mu} v^{\nu} \qquad for \quad j = 1, \\
G^{\mu_1\mu_2;\nu_1\nu_2} &= \frac{1}{2}(g_t^{\mu_1\nu_1} g_t^{\mu_2\nu_2} + g_t^{\mu_1\nu_2} g_t^{\mu_2\nu_1} - \frac{2}{3} g_t^{\mu_1\mu_2} g_t^{\nu_1\nu_2}) \quad for \quad j = 2.
\end{aligned} \tag{16}
$$

By using eqs.(10, 16) and the explicit expressions for $\Gamma^J_{\mu_1\ldots\mu_j;\rho_1\ldots\rho_J}$ given in Table 3, it is now straightforward to derive

$$
G^{\mu_1\ldots\mu_j;\nu_1\ldots\nu_j} \Gamma^J_{\mu_1\ldots\mu_j;\rho_1\ldots\rho_J} \Psi^{J^{'}}_{\nu_1\ldots\nu_j} = \delta_{JJ'} N_{J,j} u_{\rho_1\ldots\rho_j}, \tag{17}
$$

where $N_{J,j}$ is a number dependent of J and j. Combining eqs.(6),(12),(15),and (17), one arrives at eq.(7). If we normalize the current $j^{B,p,j,J}_{\rho_1\ldots\rho_j}$ appropriately, the baryonic "decay constant" $f_{j,J}$ can be written as $f_{j,J}$=$N_J f_j$

States	$\phi^{\mu_1\ldots\mu_j}_p$	j^P	$\Gamma^J_{\mu_1\ldots\mu_j;\rho_1\ldots\rho_J}$	J^P
p=K, i.e., $l_k = 0, l_K = 1$				
$\{\Lambda_{QK1}\}$	$\gamma_5 \gamma^{\mu_1}$	1^-	$\gamma_t^{\mu_1} \gamma_5 \not{D}_t$	$\frac{1}{2}^-$
			$\Gamma_{\mu_1\rho_1} \not{D}_t$	$\frac{3}{2}^-$
$\{\Sigma_{QK0}\}$	γ_t^{μ}	0^-	$\gamma_{t\mu} \not{D}_t$	$\frac{1}{2}^-$
$\{\Sigma_{QK1}\}$	$\epsilon^{\mu_1\nu\sigma\rho} \gamma_{t\nu} v^{\rho}$	1^-	$\gamma_t^{\mu_1} \gamma_5 \gamma_t^{\sigma} \not{D}_t$	$\frac{1}{2}^-$
			$\Gamma_{\mu_1\rho_1} \gamma_t^{\sigma} \not{D}_t$	$\frac{3}{2}^-$

TABLE IV. Some examples of other possible interpolating currents.

where f_j only depends on the diquark spin j and has the same value for the two states in the same doublet. Similarly, using the same method in ref. [12], we can verify

$$< 0 \mid T(j^{B,p,j,J}_{\rho_1 \cdots \rho_j}(x)\bar{j}^{B',p',j',J'}_{\rho'_1 \cdots \rho'_j}(0)) \mid 0 >\propto \delta_{BB'}\delta_{pp'}\delta_{jj'}\delta_{JJ'}. \tag{18}$$

Eqs.(7) and (18) imply that two currents with different B,p,j,J never mix in the $m_Q \to \infty$ limit. Therefore, eq.(1) are the appropriate interpolating currents for p-wave excited heavy baryons. We would like to point out that for higher (e.g., D-wave) excited states to construct interpolating currents with such properties are possible but not easy due to the presence of many different orbital states with given total orbital angular momentum of diquark.

It is well-known that the choice of interpolating currents is not unique. For example, the following interpolating currents in Table IV have also the above mentioned properties.

III. THE MASS SUM RULES

Since the sum rule for Λ_{QK1} is already presented in [13,14], we consider the sum rules for the states $\Sigma_{QK0}, \Sigma_{QK1}, \Sigma_{Qk1}, \Lambda_{Qk0}, \Lambda_{Qk1}$. To be specific, we use the interpolating currents listed in tables 3 and 4 in our calculation :

$$j_{\Sigma_{QK0}}(x) = \epsilon^{abc}[q_a^T \tau_B C \gamma_t^\mu q_b]\gamma_{t\mu} \overrightarrow{\not{D}}_t\ h_c(x) \tag{19}$$

$$j_{\Sigma_{QK1}}(x) = \epsilon^{abc}\epsilon_{\mu\nu\rho\sigma}v^\rho[q_a^T \tau_B C \gamma_t^\nu q_b]\gamma_t^\mu \gamma_t^\sigma \overrightarrow{\not{D}}_t\ \gamma_5 h_c(x), \tag{20}$$

$$j_{\Sigma_{Qk1}}(x) = \epsilon^{abc}[q_a^T \tau_B C \gamma_5\ \overrightarrow{D}_{t\mu}\ q_b]\gamma_t^\mu \gamma_5 h_c(x), \tag{21}$$

$$j_{\Lambda_{Qk0}}(x) = \epsilon^{abc}[q_a^T \tau_B C \gamma_t^\mu\ \overrightarrow{D}_{t\mu}\ q_b]h_c(x) \tag{22}$$

$$j_{\Lambda_{Qk1}}(x) = \epsilon^{abc}\epsilon_{\mu\nu\rho\sigma}v^\rho[q_a^T \tau_B C \gamma_t^\nu\ \overrightarrow{D}_t^\sigma\ q_b]\gamma_t^\mu \gamma_5 h_c(x), \tag{23}$$

where a, b, c is the color index, T denotes the transpose, C is the charge conjugate matrix, γ_t^μ is defined as above. Considering the result in the ref. [14] that the best stability of the sum rule for the current with a derivative is obtained for a=1 and b=0, we have taken a=1 and b=0 in eqs. (19-23) for the sake of simplicity. We also need

$$\bar{j}_{\Sigma_{QK0}}(x) = \epsilon^{abc}\bar{h}_c \overleftarrow{\not{D}}_t\ \gamma_{t\mu}[\bar{q}_b \gamma_t^\mu \tau_B^\dagger C \bar{q}_a^T] \tag{24}$$

$$\bar{j}_{\Sigma_{QK1}}(x) = -\epsilon^{abc}\epsilon_{\mu\nu\rho\sigma}v^\rho \bar{h}_c \gamma_5 \overleftarrow{\not{D}}_t\ \gamma_t^\sigma \gamma_t^\mu[\bar{q}_b \gamma_t^\nu \tau_B^\dagger C \bar{q}_a^T], \tag{25}$$

$$\bar{j}_{\Sigma_{Qk1}}(x) = -\epsilon^{abc}\bar{h}_c\gamma_t^\mu\gamma_5[\bar{q}_b \overleftarrow{D}_{t\mu} \gamma_5\tau_B^\dagger C\bar{q}_a^T], \tag{26}$$

$$\bar{j}_{\Lambda_{Qk0}}(x) = \epsilon^{abc}\bar{h}_c[\bar{q}_b \overleftarrow{D}_{t\mu} \gamma_t^\mu\tau_B^\dagger C\bar{q}_a^T] \tag{27}$$

$$\bar{j}_{\Lambda_{Qk1}}(x) = \epsilon^{abc}\epsilon_{\mu\nu\rho\sigma}v^\rho\bar{h}_c\gamma_t^\mu\gamma_5[\bar{q}_b \overleftarrow{D}_t^\sigma \gamma_t^\nu\tau_B^\dagger C\bar{q}_a^T], \tag{28}$$

The overlap amplitudes of the interpolating currents with the heavy baryons are defined as follows.

$$\langle 0|j_{\Sigma_{QK1}}|\Sigma_{QK1}\rangle = f_{\Sigma_{QK1}}u_{\Sigma_{QK1}} . \tag{29}$$

Similar definitions hold for other states.

In order to extract the binding energy of the p-wave heavy baryons at the leading order in the $1/m_Q$ expansion, we consider the correlation function

$$i\int e^{iqx}\langle 0|T\{j_i(x), \bar{j}_i(0)\}|0\rangle dx = \frac{1+\not{v}}{2}\Pi_i(\omega), \tag{30}$$

with $\omega = q\cdot v$.

The dispersion relation for $\Pi(\omega)$ reads

$$\Pi(\omega) = \frac{1}{\pi}\int\frac{\mathrm{Im}\Pi(s)}{s-\omega-i\epsilon}ds , \tag{31}$$

where $\mathrm{Im}\Pi(s)$ is the spectral density in the limit $m_Q\to\infty$.

At the phenomenological side we use

$$\mathrm{Im}\Pi(s) = f^2\delta(s-\Lambda) + \mathrm{Im}\Pi^{\mathrm{Pert}}(s)\theta(s-\omega_c) , \tag{32}$$

where we approximate the continuum or more higher states contribution above $m_Q+\omega_c$ with the perturbative contribution at the quark gluon level. We invoke Borel transformation with the variable ω to (31) to suppress the continuum contribution further. Finally we have

$$f^2e^{-\frac{\Lambda}{T}} = \int_0^{\omega_c}\mathrm{Im}\Pi(s)e^{-\frac{s}{T}}ds . \tag{33}$$

At the quark level the spectral density reads

$$\mathrm{Im}\Pi_{\Sigma_{QK0}}(s) = \frac{11s^7}{140\pi^3} - \frac{\langle\alpha_sG^2\rangle s^3}{96\pi^2}, \tag{34}$$

$$\mathrm{Im}\Pi_{\Sigma_{QK1}}(s) = \frac{11s^7}{35\pi^3} - \frac{\langle\alpha_sG^2\rangle s^3}{24\pi^2}, \tag{35}$$

$$\mathrm{Im}\Pi_{\Sigma_{Qk1}}(s) = \frac{s^7}{112\pi^3} + \frac{3\langle\alpha_sG^2\rangle s^3}{128\pi^2} - \frac{\langle\alpha_sG^2\rangle s^3}{32\pi^2}, \tag{36}$$

$$\mathrm{Im}\Pi_{\Lambda_{Qk0}}(s) = \frac{\tau^7}{560\pi^3} - \frac{7\langle\alpha_s G^2\rangle\tau^3}{384\pi^2}, \tag{37}$$

$$\mathrm{Im}\Pi_{\Lambda_{Qk1}}(s) = \frac{3s^7}{140\pi^3} + \frac{\langle\alpha_s G^2\rangle s^3}{96\pi^2} - \frac{\langle\alpha_s G^2\rangle s^3}{32\pi^2}, \tag{38}$$

with $\langle \frac{\alpha_s}{\pi} G^2\rangle = 0.012\mathrm{GeV}^4$ [26], where the first term of the gluon condensate in each spectral density results from the diagram with one gluon cut attached on each light quark propagator, while the second piece arises from the diagrams with one gluon cut attached on the vertex. An interesting feature of (34)-(38) is that the gluon condensate is of the opposite sign as the leading perturbative term, in contrast with the ground state baryon mass sum rules. This may be interpreted as some kind of gluon excitation since we are considering p-wave baryons. In the present case the gluon in the covariant derivative also contributes to various condensates.

To obtain the numerical results for the leading order binding energy Λ, the following formulae from the dispersion relation and quark-hardon duality is used

$$\Lambda = \frac{\frac{1}{\pi}\int_0^{\omega_c} s e^{-s/\tau}\mathrm{Im}\Pi^{\mathrm{pert}}(s)ds - \frac{d}{d1/\tau}\hat{B}^{\omega}_{\tau}\Pi^{\mathrm{cond}}(\omega)}{\frac{1}{\pi}\int_0^{\omega_c} e^{-s/\tau}\mathrm{Im}\Pi^{\mathrm{pert}}(s)ds + \hat{B}^{\omega}_{\tau}\Pi^{\mathrm{cond}}(\omega)}, \tag{39}$$

where the operator $\hat{B}$ denotes Borel transformation and $\Pi^{\mathrm{cond}}(\omega)$ denotes the condensate contribution to $\Pi(\omega)$ in (31).

With the suitable choice of τ and ω, the heavy baryon binding energy Λ is determined. All the numerical results are presented in Fig. 1. It is easy to find that the sum rules for states Σ_{QK0}, Σ_{QK1}, Σ_{Qk1}, and Λ_{Qk1} are stable in the variable region 0.2 GeV to 0.8 GeV. To find the most suitable threshold ω_c, we have tried several choices: $\omega_c = 1.4, 1.6$ and 1.8 GeV etc for each current. The final results are given at $\omega = 1.6$ GeV. In these sum rules the gluon condensate contributions are $10-30\%$ of the perturbative term in the Borel variable region from 0.3 GeV to 0.7 GeV. The lowest lying resonances dominate. As a by-product, the decay constants of these states are obtained. All the obtained binding energy Λ_i and the decay constant f_i are collected in Table V, where the variation of f_i comes from variation of Borel variable τ.

Finally, some words on the current $j_{\Lambda_{Qk0}}$ should be said. In contrast with the numerical results for the currents above, gluon condensate dominate the sum rule for the current $j_{\Lambda_{Qk0}}$ in the region with Borel variable larger than zero, so we do not give its numerical results here.

TABLE V. Binding energy of p-wave states.

-	Σ_{QK0}	Σ_{QK1}	Σ_{Qk1}	Λ_{Qk1}
Λ_i	1.35 GeV	1.32 GeV	1.30 GeV	1.28 GeV
f_i	$0.0339-0.0354\mathrm{GeV}^4$	$0.0669-0.0671\mathrm{GeV}^4$	$0.0188-0.0190\mathrm{GeV}^4$	$0.0268-0.0286\mathrm{GeV}^4$

With the renormalization-group and scheme invariant pole mass for the heavy quarks $m_c = (1.3 - 1.4)$ GeV and $m_b = (4.6 - 4.8)$GeV, we obtain the spectrum of the p-wave excited spin 1/2 doublet baryon states.

IV. DISCUSSIONS

In summary we have constructed the proper interpolating currents suitable for QCD sum rule approach for the orbitally excited heavy baryons in the framework of heavy quark effective theory. These currents are orthogonal to each other in the leading order of HQET and explore different excited states (some of them have the same quantum numbers). The mixing of these currents for the same quantum numbers occur at the order of $1/m_Q$. We want to emphasize that only in the leading order of HQET can we construct interpolating currents with such good properties. We obtain the leading order mass sum rules for the orbitally excited states with $J \leq \frac{3}{2}$ in HQET. We find the gluon condensates have opposite sign to the leading perturbative terms which may indicate the orbital excitation of the gluon fields inside the excited heavy baryons. All sum rules are stable with reasonable variations of the Borel parameter and the continuum threshold. We extract the leading order binding energy and the overlapping amplitudes, which may be used to analyze the semileptonic decays of these states. The spectrum and level spacing from our approach is consistent with that from quark model prediction [19]. Typically Λ_{QK0} lies 300 MeV higher than Λ_Q while all other excited states with $J \leq \frac{3}{2}$ lie approximately 500 MeV higher than Λ_Q. However the $1/m_Q$ correction may be significant for the excited charmed baryons. Note that the magnitudes of the binding energy we obtained are very close to the charm quark pole mass. Due to the mixing of these interpolating currents the calculation of $1/m_Q$ correction is difficult and lies out of the scope of present paper.

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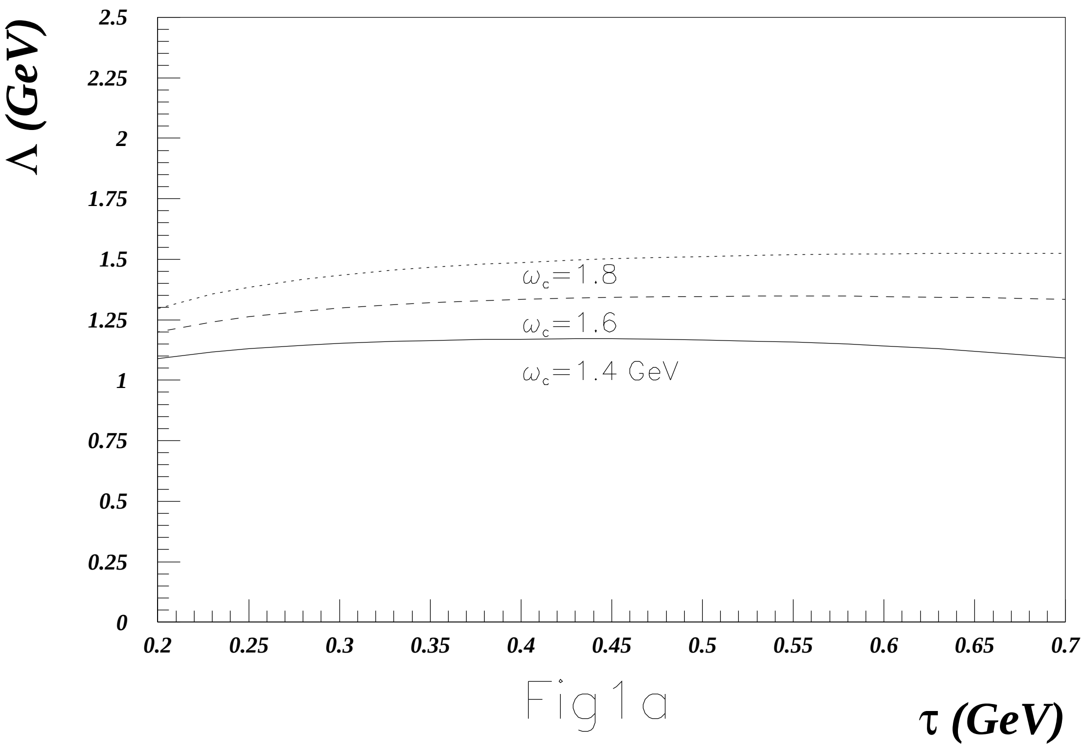
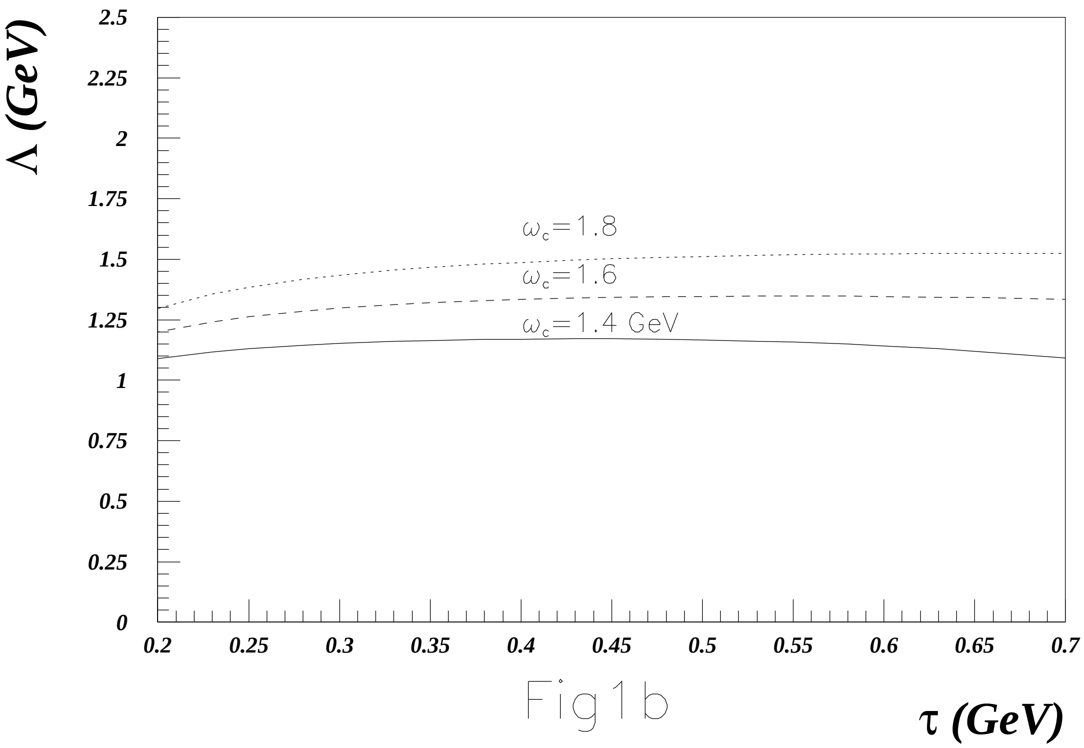
Figure Captions

FIG 1. (a) Variation of the binding energy of Σ_{QK0} with the threshold ω_c and the Borel parameter T. From top to bottom the curves correspond to $\omega_c = 1.8, 1.6, 1.4$ GeV. (b) The case for Σ_{QK1}. (c) For Σ_{Qk1}. (d) For Λ_{Qk1}.



Λ (GeV)
2.5
2.25
2
1.75
1.5
1.25
1
0.75
0.5
0.25
0
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0.55
0.6
0.65
0.7
ω_c=1.8
ω_c=1.6
ω_c=1.4 GeV
τ (GeV)

Fig1a



Λ (GeV)
2.5
2.25
2
1.75
1.5
1.25
1
0.75
0.5
0.25
0
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0.55
0.6
0.65
0.7
ω_c=1.8
ω_c=1.6
ω_c=1.4 GeV
τ (GeV)

Fig1b

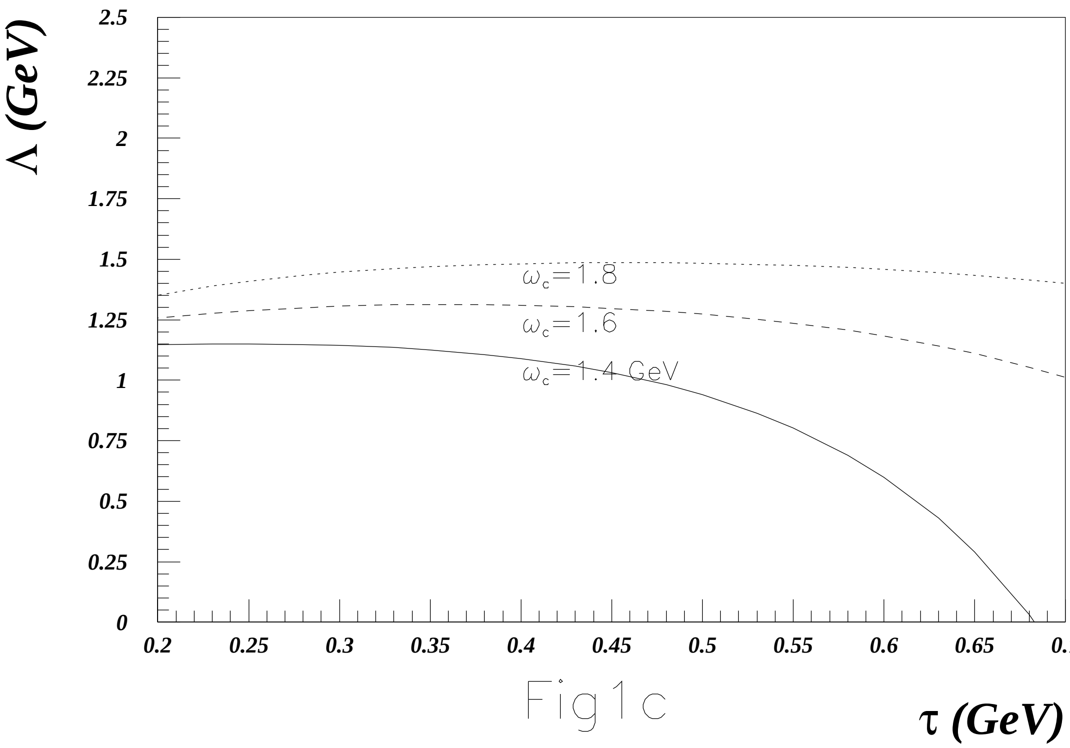
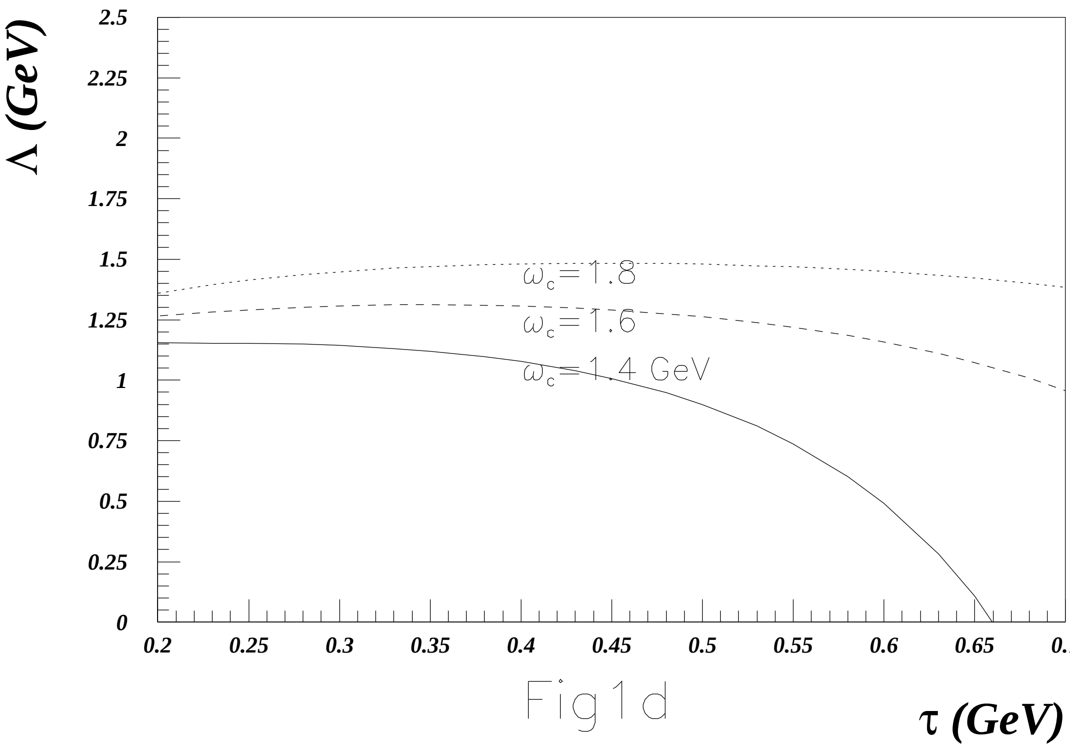

Λ (GeV)
2.5
2.25
2
1.75
1.5
1.25
1
0.75
0.5
0.25
0
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0.55
0.6
0.65
0.7
ω_c=1.8
ω_c=1.6
ω_c=1.4 GeV
τ (GeV)

Fig1c



Λ (GeV)
2.5
2.25
2
1.75
1.5
1.25
1
0.75
0.5
0.25
0
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0.55
0.6
0.65
0.7
ω_c=1.8
ω_c=1.6
ω_c=1.4 GeV
τ (GeV)

Fig1d